The Bass Model: Marketing Engineering Technical Note[1]

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Introduction

The Bass model is a very useful tool for forecasting the adoption (first purchase) of an innovation (more generally, a new product) for which no closely competing alternatives exist in the marketplace. A key feature of the model is that it embeds a "contagion process" to characterize the spread of word-of -mouth between those who have adopted the innovation and those who have not yet adopted the innovation.

The model can forecast the long-term sales pattern of new technologies and new durable products under two types of conditions: (1) the firm has recently introduced the product or technology and has observed its sales for a few time periods; or (2) the firm has not yet introduced the product or technology, but its market behavior is likely to be similar to some existing products or technologies whose adoption pattern is known. The model attempts to predict how many customers will eventually adopt the new product and when they will adopt. The question of *when* is important, because answers to this question guide the firm in its deployment of resources in marketing the innovation.

Description of the Bass model

Suppose that the (cumulative) probability that someone in the target segment will adopt the innovation by time t is given by a non-decreasing continuous function $F(t)$, where $F(t)$ approaches 1 (certain adoption) as t gets large. Such a function is depicted in Exhibit 1(a), and it suggests that an individual in the target segment will eventually adopt the innovation. The derivative of $F(t)$ is the probability density function, $f(t)$ (Exhibit 1b), which indicates the rate at which the probability of adoption is changing at time t. To estimate the unknown function $F(t)$ we specify the conditional likelihood $L(t)$ that a customer will adopt the innovation at exactly time t since introduction, given that the customer has not adopted before that time. Using the foregoing definition of $F(t)$ and $f(t)$, we can write $L(t)$ as (via Bayes's rule)

[1] This technical note is a supplement to the materials in Chapters 1, 2, and 7 of *Principles of Marketing Engineering and Analytics*, by Gary L. Lilien, Arvind Rangaswamy, and Arnaud De Bruyn (2007, 2017). Visit DecisionPro.biz for additional information. v170823

$$L(t) = \frac{f(t)}{1 - F(t)}. \qquad (1)$$

Bass (1969) proposed that $L(t)$ be defined to be equal to

$$L(t) = p + \frac{q}{N}(t). \qquad (2)$$

where

$N(t)$ = the number of customers who have already adopted the innovation by time t;

$\overline{N}$ = a parameter representing the total number of customers in the adopting target segment, all of whom will eventually adopt the product;

p = coefficient of innovation (or coefficient of external influence); and

q = coefficient of imitation (or coefficient of internal influence).

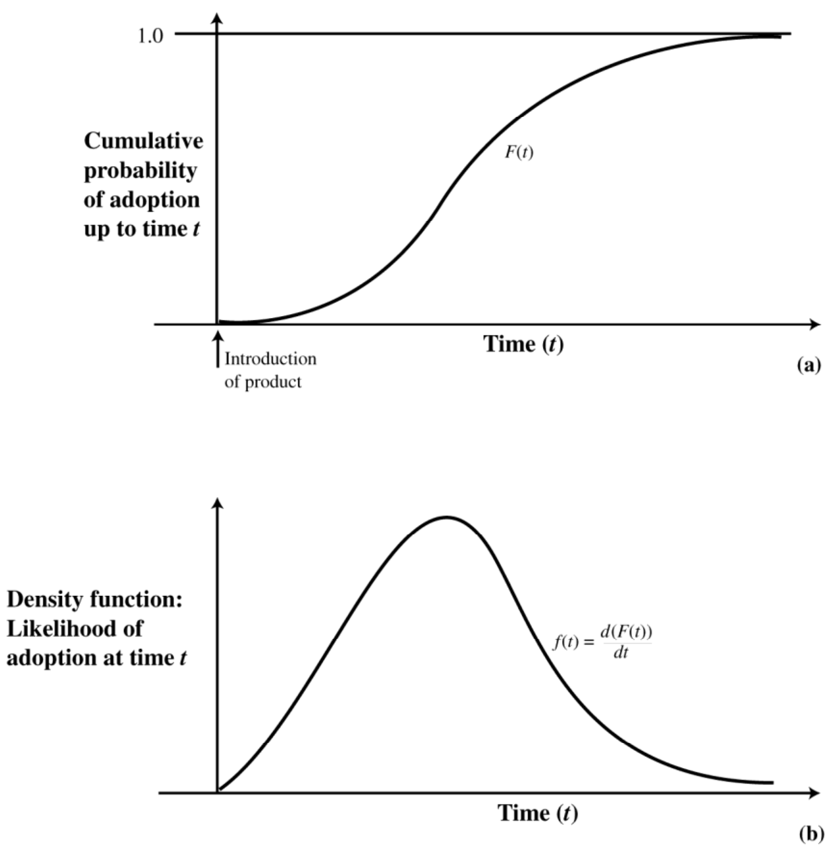

EXHIBIT 1
Graphical representation of the probability of a customer's adoption of a new product over time; (a) shows the probability that a customer in the target segment will adopt the product before time *t*, and (b) shows the instantaneous likelihood that a customer will adopt the product at exactly time *t*.

Equation (2) suggests that the likelihood that a customer in the target segment will adopt at exactly time t is the sum of two components. The first component (p) refers to a constant propensity to adopt that is independent of how many other customers have adopted the innovation before time t. The second component in Eq. (2) $[\frac{q}{\bar{N}}N(t)]$ is proportional to the number of customers who have already adopted the innovation by time t and represents the extent of favorable exchanges of word-of-mouth communications between the innovators and the other adopters of the product (imitators).

Equating Equations. (1) and (2), we get

$$f(t)=\left[p+\frac{q}{N}N(t)\right]\left[1-F(t)\right]. \tag{3}$$

Noting that $N(t)=\bar{N}F(t)$ and defining the number of customers adopting at exactly time t as $n(t)$, equal to $\bar{N}f(t)$, we get (after some algebraic manipulations) the following basic equation for predicting the sales of the product at time t:

$$n(t)=p\overline{N}+(q-p)N(t)-\frac{q}{\overline{N}}\left[N(t)\right]^2. \tag{4}$$

If $q>p$, then imitation effects dominate the innovation effects and the plot of $n(t)$ against time (t) will have an inverted U shape. On the other hand, if $q<p$, then innovation effects will dominate and the highest sales will occur at introduction and sales will decline in every period after that (e.g., blockbuster movies). Furthermore, the lower the value of p, the longer it takes to realize sales growth for the innovation. When both p and q are large, product sales take off rapidly and fall off quickly after reaching a maximum. By varying p and q, we can represent many different patterns of diffusion of innovations quite well.

An alternative (and better) way to estimate this model is to solve the differential equation, formulations of equations (1) and (2), which gives the following result.

$$F(t)=\frac{(1-e^{-(p+q)t})}{(1+\frac{q}{p}e^{-(p+q)t})} \tag{5}$$

Noting that $F(t)=\frac{N(t)}{\bar{N}}$ and if we have data on $N(t)$ for each time period t, and we know $\bar{N}$, we can directly estimate p and q from equation (5) through nonlinear least squares estimation. Alternatively, if we know p and q, we can calculate $N(t)$ for each time t.

***Generalized Bass model*:** Bass, Krishnan, and Jain (1994) propose a general form of Eq. (3) that incorporates the effects of marketing-mix variables on the likelihood of adoption:

$$f(t)=\left[p+\frac{q}{N}N(t)\right]\left[1-F(t)\right]x(t), \tag{6}$$

where $x(t)$ is a function of the marketing-mix variables in time period t. One way to specify $x(t)$ is as follows (assuming that the marketing mix variables of interest are advertising and pricing).

$$x(t)=1+\alpha\frac{\left[P(t)-P(t-1)\right]}{P(t-1)}+\beta Max\left\{0,\frac{\left[A(t)-A(t-1)\right]}{A(t-1)}\right\} \tag{7}$$

α = coefficient capturing the percentage increase in diffusion speed resulting from a

1% decrease in price,

$P(t)$ = price in period t,

β = coefficient capturing the percentage increase in diffusion speed resulting from a 1% increase in advertising,

$A(t)$ = advertising in period t,

The continuous version of equation (7) is given below, and is the one implemented in Marketing Engineering.

$$x(t) = t + Ln\left[\frac{\mathrm{Pr}(t)}{\mathrm{Pr}(1)}\right]\beta_1 + Ln\left[\frac{\mathrm{Ad}(t)}{\mathrm{Ad}(1)}\right]\beta_2 \qquad (8)$$

Equation (6) implies that by increasing marketing effort, a firm can increase the likelihood of adoption of the innovation — that is, marketing effort speeds up the rate of diffusion of the innovation in the population. For implementing the model, we can measure marketing effort relative to a base level indexed to 1.0. Thus if advertising at time t is double the base level, $x(t)$ will be equal to 2.0.

Estimating the Bass model parameters

There are several methods to estimate the parameters of the Bass model. These methods can be classified based on whether they rely on historical sales data or judgment for calibrating the model. Linear and nonlinear regression can be used if we have historical sales data for the new product for a few periods (years). Judgmental methods include using analogs or conducting surveys to determine customer purchase intentions. Perhaps the simplest way to estimate the model is via nonlinear regression. By discretizing the model in Eq. (3) and multiplying both sides by ($\overline{N}$) we get:

$$n(t) = \left[p + \frac{q}{\overline{N}}N(t-1)\right]\left[\overline{N} - N(t-1)\right]. \qquad (9)$$

On simplification, this becomes:

$$n(t) = p\overline{N} + (q-p)N(t-1) + \frac{q}{\overline{N}}[N(t-1)]^2$$

which is equivalent to the following linear regression model:

$$n(t) = a + bN(t-1) + c[N(t-1)]^2: \qquad (10)$$

Given at least four observations of $N(t)$ we can use linear regression to estimate parameter values ($\overline{N}, p, q$) to minimize the sum of squared errors. There are more sophisticated approaches for estimating the parameters of the Bass model, including maximum likelihood estimation (Srinivasan and Mason 1986) and Hierarchical Bayes estimation (Lenk and Rao 1990). For the latter approaches, we need to know the time at which the product was introduced into the market, something that could be difficult to determine for some older products. In marketing Engineering, we have implemented the recommendations given in Srinivasan and Mason (1986), and Jiang, Bass and Bass (2006), which taken together provides the following formula for estimating the Bass model, the generalized Bass model, and also allows for the possibility that the data for estimation were collected a few periods after the product/technology was introduced into the market.

$$F(t) = \frac{(1-e^{-(p+q)(X(t)+\,\tau-X(0))})}{(1+\frac{q}{p}e^{-(p+q)(X(t)+\,\tau-X(0))})} \qquad (11)$$

where *X(t)* is cumulative marketing effort up to time *t*, and *X(0)* is the initial marketing effort. If marketing effort in each period, *x(t),* is equal to 1 for all *t*, then equation (11) represents the standard Bass model; otherwise, it represents the generalized Bass model. Thus, for the basic Bass model *X(t)* = *t*, and *X(0)* = 0. τ is the time elapsed from product introduction to the start time of the data series. If τ is not equal to 0, as a simplification, we assume that the marketing effort is held constant from the time of product introduction to the time where the data series starts.

Using Bass Model Estimates for Forecasting

Once we determine the parameter values by estimating or by using analogs, we can put these values into a spreadsheet to develop forecasts (Exhibit 2). The software has built-in options for sales forecasting using estimates obtained either from the nonlinear least squares method (if there is sufficient market data for estimation) or by directly selecting *p* and *q* from analogous products.

Sales in each period (reported every quarter)

Quarter	Sales	Cumulative sales
0	0	0
1	160	160
4	425	1,118
8	1,234	4,678
12	1,646	11,166
16	555	15,106
20	78	15,890
24	9	15,987

Example computations (from Equation 7.9)

Sales in Quarter 1 =

0.01 * 16,000 + (0.41 – 0.01) * 0 – (0.41 / 16,000) * $(0)^2$ = 160

Sales in Quarter 2 =

0.01 * 16,000 + (0.41 – 0.01) * 160 – (0.41 / 16,000) * $(160)^2$ = 223.35

Sales in Quarter 4 =

0.01 * 16,000 + (0.41 – 0.01) * 692.9 – (0.41 / 16,000) * $(692.9)^2$ = 424.8

EXHIBIT 2

Example computations showing how to use the Bass model to forecast the sales of an innovation (here, room temperature control unit). The computations are based on the estimated values of *p*=0.1 and *q*=0.41, and market potential ($\bar{N}$)=16,000 units (in thousands). This table is for illustrative purposes only. Marketing Engineering uses equation (11) for both estimation and forecasting.

For forecasting, we recommend that you determine $\bar{N}$ via an external procedure (e.g., survey of long-term purchase intentions) and use nonlinear regression given in Eq. (6) for estimating *p* and *q*, if you have some past data for the focal technology. If you have data on marketing effort, then it is best if you use Generalized Bass Model for estimation. In some cases, even if you do not have actual data on marketing efforts, you might have useful judgmental data. When you don't have any data for the focal product, a promising approach for forecasting using the Bass Model or the Generalized Bass Model is to rely on analog products for which *p* and *q* values are known. In these cases, we may sometimes have to deal with situations where some adoptions have already taken place at the start of the forecasting period. In such cases, we use the following formula to calculate τ in order to apply equation 11 for forecasting.

$$\tau = -\frac{Ln\left\{\frac{1-F(\tau)}{1+F(\tau)\frac{q}{p}}\right\}}{(p+q)} \qquad (12)$$

where $F(\tau)$ is calculated from the initial non-zero adoptions at the start of the forecasting analysis. If number of adoptions at the start of the forecasting period is equal to a, and market potential at start is $\bar{N}$, then $F(\tau) = \frac{a}{\bar{N}}$. With $F(\tau), p\ and\ q$ known, we can compute τ from equation (12)

The Bass model has been extensively used for understanding how successful innovations have diffused through the population. In applying the Bass model, especially in forecasting contexts, it is important to recognize its limitations. Most past data (from analogs) describe how successful innovations have diffused through the population, but do not account for their chances of success. Thus, such data would predict favorable forecasts for any new product, resulting in a success bias in the forecasts. To minimize such a bias, one must incorporate the probability of product failure in the model. Unfortunately, we currently know little about the sales patterns of innovations that failed. Another limitation of the Bass model is that we can estimate its parameters well from data only after making several observations of actual sales. However, by this time the firm has already made critical investment decisions. While the use of analogs can help firms make forecasts before introducing an innovation into the market, the choice of a suitable analog is critical and requires careful judgment.

Some Extensions of the Bass model

The Bass model makes several key assumptions. We can relax several of these assumptions by using more sophisticated models as summarized below:

- *The market potential ($\bar{\mathrm{N}}$) remains constant*: This assumption is relaxed in models in which $\bar{\mathrm{N}}$ is a function of price declines, uncertainty about technology performance, and growth of the target segment. The software includes an option to specify the growth rate of the target segment, either due to intrinsic growth of the target population, and/or due to price declines.

  (1) Intrinsic growth in market potential: If market size is growing over time, we can compute it as follows:

  $$\bar{N}(t) = \bar{N}(t-1) + \bar{N}(t-1) * \frac{g}{100}$$

  where $\bar{N}(0)$ = Total Market Potential at the start and g is Market Growth Rate in percentage, and *t* is time since start of diffusion process.

  (2) Market growth due to price declines: Let price elasticity of market potential be m_p(we will define this in terms of increase in market potential for a *decrease* in price). We can also

assume that the price change is initiated at the start of time period *t*. Here is the mathematical representation of m_p (we assume m_p is provided as a percentage):

$$\frac{m_p}{100} = \frac{\frac{d\overline{N}(t)}{\overline{N}(t)}}{\frac{dPr(t)}{Pr(t)}}$$

Converting this into a discrete form:

$$\frac{\Delta\overline{N}(t)}{\overline{N}(t-1)} = \left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(t)}{Pr(t-1)}\right]$$

$$\Delta\overline{N}(t) = \overline{N}(t-1)\left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(t)}{Pr(t-1)}\right]$$

$$\overline{N}(t) = \overline{N}(t-1) + \overline{N}(t-1)\left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(t)}{Pr(t-1)}\right]$$

If we have both market growth and price change at any time *t*, we use the following approach to calculate market potential cumulatively starting from time *t* = 1, as illustrated in the following table for the first few periods. .

Time	**Market growth**	**Growth from price change**	**Market potential $\overline{N}(t)$, including both intrinsic growth and growth due to price declines.**
1			$\overline{N}(1) = \overline{N}(0)$
2	$\frac{g}{100}$	$\overline{N}(1)\left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(2)}{Pr(1)}\right]$	$\overline{N}(2) =$ $\overline{N}(1) + \overline{N}(1)\left\{\frac{g}{100} + \left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(2)}{Pr(1)}\right]\right\}$
3	$\frac{g}{100}$	$\overline{N}(2)\left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(3)}{Pr(2)}\right]$	$\overline{N}(3) =$ $\overline{N}(2) + \overline{N}(2)\left\{\frac{g}{100} + \left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(3)}{Pr(2)}\right]\right\}$
4	$\frac{g}{100}$	$\overline{N}(3)\left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(4)}{Pr(3)}\right]$	$\overline{N}(4) =$ $\overline{N}(3) + \overline{N}(3)\left\{\frac{g}{100} + \left[\frac{m_p}{100}\right]\left[\frac{\Delta Pr(4)}{Pr(3)}\right]\right\}$

A key assumption we make is that market penetration, *F(t)*, and market potential, $\overline{N}(t)$, evolve independently and the cumulative number of adopters at any time *t* is just the product of these two factors, *F(t)*$\overline{N}(t)$. There are other formulations that are desirable where, for example, $\overline{N}(t)$ could influence *F(t)*, but our approach should work reasonably well in situations where growth in $\overline{N}(t)$ is gradual.

- *The marketing strategies supporting the innovation do not influence the adoption process*: Considerable research has been devoted to incorporating the impact of marketing variables, particularly price, advertising, and selling effort. We described the generalized Bass model, which represents one way to relax this assumption.
- *The customer decision process is binary (adopt or not adopt)*: This assumption is relaxed in several models that incorporate multistage decision processes in which the customer goes from one phase to another over time: awareness → interest → adoption → word of mouth.
- *The value of q is fixed throughout the life cycle of the innovation*: One would, however, expect interaction effects (e.g., word of mouth) to depend on adoption time, being relatively strong during the early and late stages of a product's life cycle. This assumption is relaxed in models that incorporate a time-varying imitation parameter.

- *Uniform mixing, i.e., everyone can come into contact with everyone else*. One way to relax this assumption is by incorporating the social structure of connections among the members of the target group. An appealing structure to include is the "small world network" with both "close" and "distant" ties among members.
- *Imitation always has a positive impact (i.e., the model allows only for interactions between innovators and non-innovators who favor the innovation)*: Several models are available that allow for both positive and negative word of mouth. When word-of-mouth effects are likely to be positive (e.g., "sleeper" movies such as *Ghost*), it may be wise to gradually ramp up marketing expenditures, whereas when word-of-mouth effects are likely to be negative (e.g., the "mega-bomb" movie *Waterworld*), it may be better to advertise heavily initially to generate quick trials before the negative word of mouth significantly dampens sales.
- *Sales of the innovation are considered to be independent of the adoption or non-adoption of other innovations*: Many innovations depend on the adoption of related products to succeed. For example, the adoption of multimedia software depends on the adoption of more powerful PCs. Likewise such innovations as wide area networks and electronic commerce complement each other and have to be considered jointly to predict their sales. Several models are available for generating forecasts for products that are contingent on the adoption of other products.
- *There is no repeat or replacement purchase of the innovation*: There are several models that extend the Bass model to forecast purchases by both first-time buyers and by repeat buyers.

Summary

The Bass model provides a conceptually appealing and mathematically elegant structure to explain how a new technology or product diffuses through a target population of customers. The model can be used for long-term forecasting of the adoption of an innovation. Such forecasts are not only important for the firm introducing the innovation (e.g., Apple's introduction of iPod), but also for other companies that make related products that complement, or substitute for, the innovation (e.g., record labels, speaker makers). Over the years, many of the assumptions of the original Bass model have been relaxed to provide us with a rich framework within which to model the time path of new product adoptions. Here, we gave an outline of the model and its estimation and use in forecasting applications. One of the important benefits of the Bass model is for forecasting the diffusion of a focal product by using the parameters of the diffusion process for analogous products.

References

Bass, Frank M. (1969), "A new product growth model for consumer durables," *Management Science,* Vol. 15, No. 5, p. 215-227.

Lenk, Peter J. and Ambar Rao (1990), "New Models from Old: Forecasting Product Adoption By Hierarchical Bayes Procedure," *Marketing Science*, Vol. 9, No. 1, 42-53.

Jiang, Zhengrui, Frank M. Bass, and Portia Isaacson Bass (2006), "Virtual Bass Model and the left-hand data-truncation bias in diffusion of innovation studies," *International Journal of Research in Marketing*, Vo, 23, p. 93-106.

Srinivasan, V., and Charlotte H Mason (1986), "Nonlinear Least Squares Estimation of New Product Diffusion Models," *Marketing Science*, Vol. 5, No. 2 (Spring), p. 169-178.